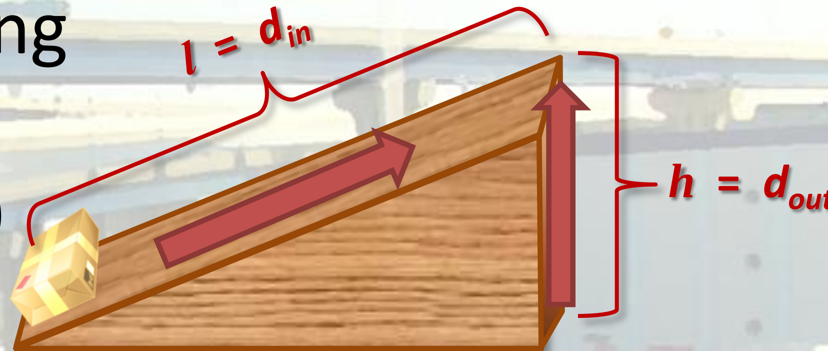
Inclined Planes

How Does an Incline Work?

- An inclined plane is a ramp.
- You slide an object up along the length of the ramp.
    - This is the work input (W_{in})
    - $W_{in} = F_{in} \times d_{in}$

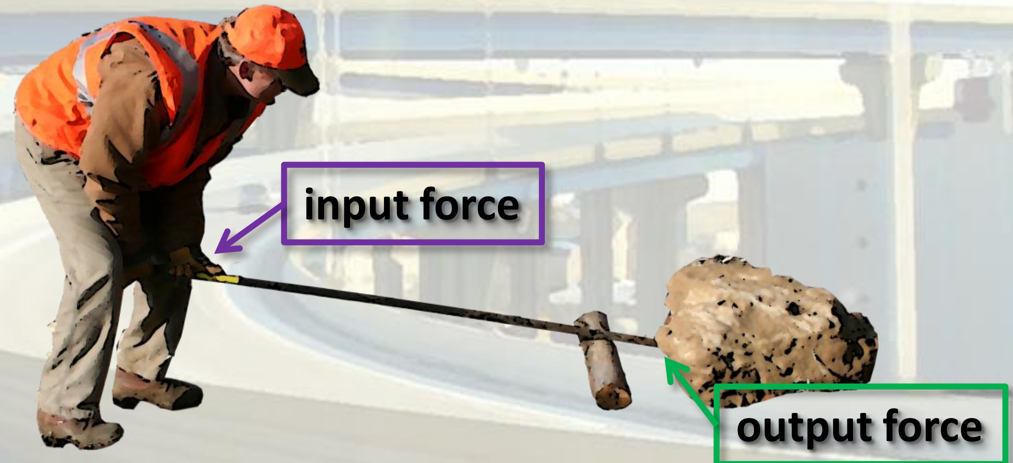


- The work actually accomplished is lifting the object up the height of the ramp from the bottom to the top.
    - This is the work output (W_{out})
    - $W_{out} = F_{out} \times d_{out}$

REVIEWING CONCEPTS

Mechanical Advantage

- The applied force to a machine is called the **input force**.
- The resulting force from the machine is called the **output force**.

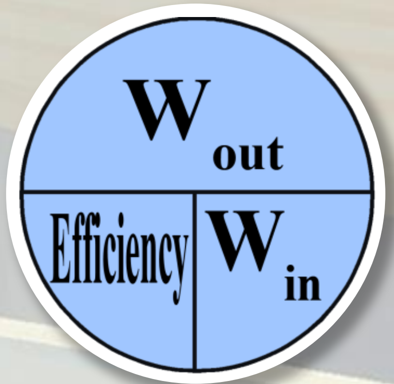


- The **mechanical advantage** is the ratio of the output force to the input force.
  - *mechanical advantage* = $\frac{\textit{output force}}{\textit{input force}}$
  - $MA = \frac{F_{out}}{F_{in}}$

Efficiency

- Work output (W_{out}) is all useful work. It is what you are trying to accomplish.
- Work input (W_{in}) is useful work **plus** work done overcoming friction.
- The more work a machine requires in order to overcome friction, the less efficient it is.
- Efficiency is the percentage of useful work out of the total work done.

$$\text{Efficiency} = \frac{W_{out}}{W_{in}} \times 100\ \%$$




BACK TO THE INCLINE

Incline Mathematical Relationships

- Ideally, if no work is wasted overcoming friction, the work input is equal to the work output.
  - $W_{in} = W_{out}$
  - This means $F_{in} \times d_{in} = F_{out} \times d_{out}$
- Rearranging the equation results in the following:
  - $\frac{d_{in}}{d_{out}} = \frac{F_{out}}{F_{in}}$

Mechanical Advantage of an Incline

- The mechanical advantage is the ratio of the output force to the input force.
- Since $\frac{d_{in}}{d_{out}} = \frac{F_{out}}{F_{in}}$ they are both the mechanical advantages.

- The dimensions of the incline are not affected by friction.
  - The ratio of the input distance to the output distance is the ideal mechanical advantage.
  - IMA = $\frac{d_{in}}{d_{out}}$
- The input force is increased by friction, reducing the mechanical advantage.
  - The ratio of the output force to the input force is the actual mechanical advantage.
  - AMA = $\frac{F_{out}}{F_{in}}$

Sample Problem

A 980 N crate is pushed with a force of 196 N along 2.1 m plank to a shelf 0.3 m high.

- What is the ideal mechanical advantage?

$$IMA = \frac{d_{in}}{d_{out}} = \frac{2.1m}{0.3m} = 7$$

- What is the actual mechanical advantage?

$$AMA = \frac{F_{out}}{F_{in}} = \frac{980\ N}{196\ N} = 5$$

- What is the work input?

$$W_{in} = F_{in} \times d_{in} = (196\ \text{N})(2.1\ \text{m}) = 411.6\ \text{Nm} = 411.6\ \text{J}$$

- What is the work output

$$W_{out} = F_{out} \times d_{out} = (980\ \text{N})(0.3\ \text{m}) = 294\ \text{Nm} = 294\ \text{J}$$

Same Sample Problem Continued

A 980 N crate is pushed with a force of 196 N along 2.1 m plank to a shelf 0.3 m high.

- How much work was done overcoming friction?

  $W_f = W_{in} - W_{out} = 411.6 \text{ J} - 294 \text{ J} = 117.6 \text{ J}$

- What is the machines efficiency?

  $\text{Efficiency} = \frac{W_{out}}{W_{in}} \times 100\ \% = \frac{294 \text{ J}}{411.6 \text{ J}} \times 100\ \% = 71.4\ \%$

- What is the force needed to overcome friction?

  $F_f = \frac{W_f}{d_{in}} = \frac{117.6 \text{ J}}{2.1 \text{ m}} = 56 \text{ N}$

- If there were no friction, what force would ideally be needed to slide the crate?

  $F_{in} = \frac{F_{out}}{IMA} = \frac{980 \text{ N}}{7} = 140 \text{ N}$