

The Ideal Gas Law, Molar Mass, and Density

There are several relationships between the temperature, pressure, the number of moles and the volume of gases. Boyle's law says at constant temperature, the volume and pressure of a sample of gas are inversely proportional [$V \propto 1/P$]. Charles law says at constant pressure, the volume and temperature of a sample of gas are directly proportional [$V \propto T$]. Gay-Lussac's law says at constant volume, the temperature and pressure of a sample of gas are directly proportional [$T \propto P$]. Finally Avogadro's law says at constant temperature and pressure, the volume of a gas is directly proportional to the number of moles [$V \propto n$]. That's a lot of laws. Fortunately, they can be expressed together as one relationship known as the ideal gas law:

$$PV \propto nT \text{ or } PV = nRT$$

R is the universal gas constant. It can be derived as follows:

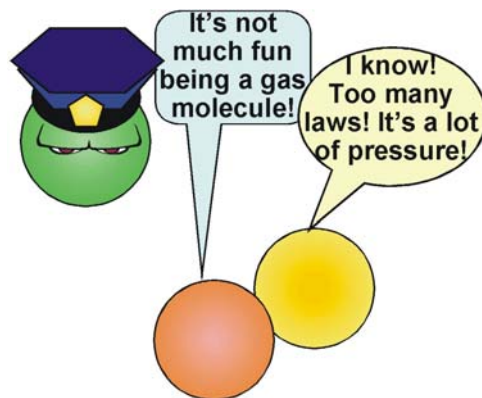
If $PV = nRT$

- $R = \frac{PV}{nT}$
- at STP, when $n = 1$, $V = 22.4L$, $T = 273K$, $P = 1atm$
- $R = \frac{(1atm)(22.4L)}{(1mol)(273K)} = 0.0821 \frac{L \cdot atm}{mol \cdot K}$

The molar mass and density of a gas can be determined from the ideal gas law.

$PV = nRT$; If $m = \text{mass}$ and $M = \text{molar mass}$, $n = \frac{m}{M}$

- $PV = \frac{mRT}{M}$
- $M = \frac{mRT}{PV}$ but $D = \frac{m}{V}$ so $M = \frac{DRT}{P}$
- $D = \frac{MP}{RT}$



The effect of pressure on gases.

Using the gas constant and the ideal gas law, it is possible to determine the value of any of the four variables knowing the other three. Mass can even be used as one of the variables since it has a relationship with moles.

Sample Problem

What is the volume of 6.06 g of hydrogen at 27°C and 1.50 atm?

$$PV = nRT \quad \therefore \quad V = \frac{nRT}{P}$$

$$6.06g \left(\frac{1mol}{2.02g} \right) = 3.00mol$$

$$V = \frac{(3.00mol)(0.0821 \frac{L \cdot atm}{mol \cdot K})(300.K)}{1.50atm} = 49.3L$$

Sample Problem 1

What is the molar mass of a gas that has a density of 2.16 g/L at 15°C and 3.00 atm?

$$M = \frac{(2.16 \frac{g}{L})(0.0821 \frac{L \cdot atm}{mol \cdot K})(288K)}{3.00atm} = 17.0 \frac{g}{mol}$$

Sample Problem 2

What is the density of methane (CH₄) at 100.°C and 2.00 atm?

$$D = \frac{(16.0 \frac{g}{mol})(2.00atm)}{(0.0821 \frac{L \cdot atm}{mol \cdot K})(373K)} = 1.04 \frac{g}{L}$$

Answer the questions below using the procedures illustrated on the previous page.

1. What is the volume of 20.0 mol of gas at 20.0°C and 200. kPa? [*NOTE*: 1 atm = 101.3 kPa]
2. How many grams of oxygen occupy 150. mL at 25.0°C and 0.250 atm?
3. What is the density of nitrogen at -25.0°C and 4.25 atm?
4. What is the molar mass of a gas with a density of 4.03 g/L at -73°C and 1,140 torr? [*NOTE*: 1 atm = 760. torr]
5. At what pressure will 99.0 g of steam (H₂O) occupy 61.6 L at 125°C?